1. In a group of 100 sports car buyers, 40 bought alarm systems, 30 purchased bucket seats, and 20 purchased an alarm system and bucket seats. If a car buyer chosen at random bought an alarm system, what is the probability they also bought bucket seats?

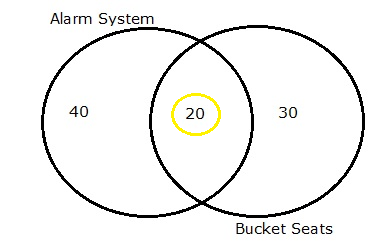
Ans:

Step 1: Figure out P(A). It’s given in the question as 40%, or 0.4.

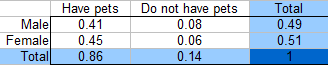
Step 2: Figure out P(A∩B). This is the intersection of A and B: both happening together. It’s given in the question 20 out of 100 buyers, or 0.2.

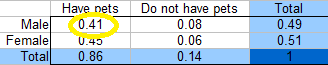
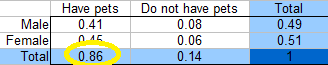
Step 3: Insert your answers into the formula:  
P(B|A) = P(A∩B) / P(A) = 0.2 / 0.4 = 0.5.

The probability that a buyer bought bucket seats, given that they purchased an alarm system, is 50%.

[](http://www.statisticshowto.com/wp-content/uploads/2012/10/venn-diagram-of-conditional-probability.png)

*Venn diagram showing that 20 out of 40 alarm buyers purchased bucket seats.*

**Example 2:**This question uses the following contingency table:  
[](http://www.statisticshowto.com/wp-content/uploads/2012/10/conditional-contingency.png)  
  
  
What is the probability a randomly selected person is male, given that they own a pet?  
Step 1: Repopulate the formula with new variables so that it makes sense for the question (optional, but it helps to clarify what you’re looking for). I’m going to say M is for male and PO stands for pet owner, so the formula becomes:  
P(M|PO) = P(M∩PO) / P(PO)

Step 2: Figure out P(M∩PO) from the table. The intersection of male/pets (the intersection on the table of these two factors) is 0.41.  
[](http://www.statisticshowto.com/wp-content/uploads/2012/10/conditional-contingency3.png)  
  
  
Step 3: Figure out P(PO) from the table. From the total column, 86% (0.86) of respondents had a pet.  
[](http://www.statisticshowto.com/wp-content/uploads/2012/10/conditional-contingency2.png)

Step 4: Insert your values into the formula:  
P(M|PO) = P(M∩PO) / P(M) = 0.41 / 0.86 = 0.477, or 47.7%.

3. The probability of 7 when [***rolling two die***](https://www.cut-the-knot.org/Probability/Probabilities.shtml#TwoDieTable) is 1/6 (= 6/36) because the [***sample space***](https://www.cut-the-knot.org/Probability/SampleSpaces.shtml) consists of 36 ***[equiprobable](https://www.cut-the-knot.org/Probability/Probabilities.shtml" \l "equiprobable)***[***elementary outcomes***](https://www.cut-the-knot.org/Probability/Dictionary.shtml#elementary) of which 6 are [***favorable***](https://www.cut-the-knot.org/Probability/Dictionary.shtml#favorable) to the [***event***](https://www.cut-the-knot.org/Probability/Dictionary.shtml) of getting 7 as the sum of two die. Denote this event A: P(A) = 1/6.

Ans:

Consider another event B which is having at least one 2. There are still 36 elementary outcomes of which 11 are favorable to B; therefore, P(B) = 11/36. We do not know whether B happens or not, but this is a legitimate question to inquire as to what happens if it does. More specifically, what happens to the probability of A under the assumption that B took place?

The assumption that B took place reduces the set of possible outcomes to 11. Of these, only two - 25 and 52 - are favorable to A. Since this is reasonable to assume that the 11 elementary outcomes are equiprobable, the probability of A under the assumption that B took place equals 2/11. This probability is denoted P(A|B) - the probability of A assuming B: P(A|B) = 2/11. More explicitly P(A|B) is called the *conditional probability* of A assuming B. Of course, for any event A, P(A) = P(A|Ω), where, by convention, [***Ω***](https://www.cut-the-knot.org/Probability/Dictionary.shtml) is the *universal event* - the whole of the sample space - for which all available elementary outcomes are favorable.

We see that, in our example, P(A|B) ≠ P(A). In general, this may or may not be so.

Retracing the steps in the example,

P(A|B) = P(A∩B) / P(B),

and this is the common definition of the *conditional probability*. The formula confirms to the earlier definitions:

P(A|Ω) = P(A∩Ω) / P(Ω) = P(A)

since P(Ω) = 1 and A∩Ω = A (because A⊂Ω.)

For another example, let's look at the events associated with rolling a dice in the form of [***octahedron***](https://www.cut-the-knot.org/do_you_know/polyhedra.shtml), a shape with eight faces. The sample space naturally consists of 8 equiprobable outcomes:

Ω = {1, 2, 3, 4, 5, 6, 7, 8}.

Let A be the event of getting an odd number, B is the event getting at least 7. Then P(A) = 4/8 = 1/2, P(B) = 2/8 = 1/4.A∩B = {7}, so that

P(A|B) = P(A∩B)/P(B) = 1/2 = P(A).

Observe also that

P(B|A) = P(A∩B)/P(A) = 1/4 = P(B).

On the other hand, define

A+ = {1, 2, 3, 5, 7} and  
A- = {3, 5, 7}.

Then

P(B|A+) = P(A+∩B)/P(A+) = 1/5 < 1/4 = P(B),

whilst

P(B|A-) = P(A-∩B)/P(A-) = 1/3 > 1/4 = P(B).

So we see that, in general, there is no definite relationship between the probability P(B) and the conditional probability P(B|A). They may be equal, or one of them may be greater than the other. In the former case the events are said to be *independent*.

Conditional probabilities arise naturally (and prove useful) in the investigation of experiments run repeatedly where an outcome of a trial may affect the outcomes of the subsequent trials. *Drawing without replacement* is one class of such experiments. Making a selection by drawing a straw or a match from a bunch of apparently equal items is a common practice throughout the world [[***Falk***](https://www.cut-the-knot.org/Probability/Conditionalprobability.shtml#Falk), 2.3.3].

For a group of six kids, six identical-looking matches are used. An uninvolved person secretly breaks one match at its lower end and then holds all six in his/her palm, so that the lower ends are hidden, and only six upper ends are visible arranged evenly.

The first child randomly draws a match. If the match is the broken one, the child is selected, and the drawing procedure stops. If not, the second child draws a match, and so on. The drawing procedure terminates when the broken match is selected.

Let us denoted the six children by their ordinal numbers in the draw: 1, 2, ..., 6. (The randomness, or rather arbitrariness of the order can be assured by an entertaining activity of [***drawing shuttles***](https://www.cut-the-knot.org/Curriculum/Algebra/Shuttles.shtml).) For each child, compute the probability that he or she will be the selected person.

The probability for the first child is obviously 1/6 because there are 6 matches with equal chances of being drawn. The probability that the first child will not be selected is 5/6. If it's not, the remaining 5 matches have equal chances to be drawn by the second child who, therefore, has the probability of 1/5 of being selected on the second drawing. But this probability is conditional on the failure of the first child to win the selection. In other words, the conditional probability P(B|A) = 1/5. where A is the event of the first child drawing the broken match, and B is that for the second child. The probability of the second child being drawn is then

|  |  |
| --- | --- |
| P(B) | = P(B∩A) + P(B∩A) |
|  | = P(B∩A) |
|  | = P(B|A)×P(A) |
|  | = 1/5 × 5/6 |
|  | = 1/6. |

This is because P(A∩B) = 0, as only one child could be selected. All individual child selections are [***mutually exclusive events***](https://www.cut-the-knot.org/Probability/Dictionary.shtml#incompatible).

The probability that neither of the first two children is selected 5/6·4/5 = 4/6, i.e., P(A∩B) = 4/6. If C is the event of the third child being selected then

|  |  |
| --- | --- |
| P(C) | = P(C∩(A∩B)) |
|  | = P(C|A∩B)×P(A∩B) |
|  | = 1/4 × 4/6 |
|  | = 1/6. |

The similar reasoning applies to the remaining children so that all six of them have an equal chance to draw the broken match. The procedure is quite fair, although, at first sight, the result is counterintuitive.

In describing the survival rate and life expectancy in a certain population, let AN denote the event of reaching the age of N years and P(N) = (AN) be the corresponding probability. In other words, P(N) stands for the probability of a new-born to reach the age of N years. We are given that

P(50) = .913,  
P(55) = .881,  
P(65) = .746.

This information suggests several questions. For example, what is the probability of a 50 years old man to reach the age of 55, i.e. what is P(55|50) = P(A55|A50)? Since obviously A55∩A50 = A55, we have by definition,

P(55|50) = P(A55∩A50)/P(A50) = P(A55)/P(50) ≈ .965.

A probability that a 50 years old will die within 5 years is then a rather comforting 1 - .965 = .035. However, as it should, the probability of dying within the next 5 years grows with age. So if, for example, the probability that a man who just turned 65 will die within 5 years is .16, what is the probability for a man to survive till his 70th birthday, i.e., what is P(70)?

As before, P(70|65) = P(70)/P(65) so that P(70) = P(65)·P(70|65), but

P(70|65) = 1 - .16 = .84.

Therefore,

P(70) = P(65)·P(70|65) = .746·.84 ≈ .627.

3. Suppose a voter poll is taken in three states. In state A, 50% of voters support the liberal candidate, in state B, 60% of the voters support the liberal candidate, and in state C, 35% of the voters support the liberal candidate. Of the total population of the three states, 40% live in state A, 25% live in state B, and 35% live in state C. Given that a voter supports the liberal candidate, what is the probability that she lives in state B?

By Bayes's formula,

*P(Voter lives in state B|Voter supports liberal candidate) =*

*P(Voter supports liberal candidate|Voter lives in state B)P(Voter lives in state B)/*

*(P(Voter supports lib. cand.|Voter lives in state A)P(Voter lives in state A) +*

*P(Voter supports lib. cand.|Voter lives in state B)P(Voter lives in state B) +*

*P(Voter supports lib. cand.|Voter lives in state C)P(Voter lives in state C))*

= (0.60)\*(0.25)/((0.50)\*(0.40) + (0.60)\*(0.25) + (0.35)\*(0.35))

= (0.15)/(0.20 + 0.15 + 0.1225) = 0.15/0.4725 = 0.3175.

The probability that the voter lives in state B is approximately 0.32.